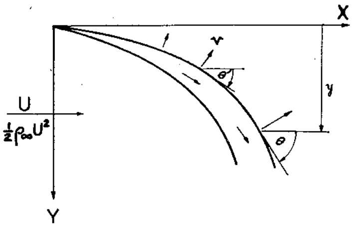

Fig. 1 Sail geometry and notation

in the equilibrium equation of the sail

$$p = T(d\theta/ds) \tag{7}$$

where s is the arc length of the sail and T the tension in the sail per unit depth.

First neglect the centrifugal correction and correct the shape of the sail studied by Daskin and Feldman.[4] For simplicity assume that σ is independent of p. Thus one gets immediately

$$\frac{2q_\infty}{T} s = \int_{\theta_L}^{\theta} \frac{d\theta}{\sin^2\theta} + 2\sigma \int_{\theta_L}^{\theta} \frac{d\theta}{\sin\theta} = (\cot\theta_L - \cot\theta) + 2\sigma \ln(\tan\tfrac{1}{2}\theta/\tan\tfrac{1}{2}\theta_L) \tag{8}$$

$$\frac{2q_\infty}{T} x = \int_{\theta_L}^{\theta} \cos\theta \frac{2q_\infty}{T}\frac{ds}{d\theta} d\theta = \left(\frac{1}{\sin\theta_L} - \frac{1}{\sin\theta}\right) + 2\sigma \ln(\sin\theta/\sin\theta_L) \tag{9}$$

$$\frac{2q_\infty y}{T} = \int_{\theta_L}^{\theta} \sin\theta \frac{2q_\infty}{T}\frac{ds}{d\theta} d\theta = \ln(\tan\tfrac{1}{2}\theta/\tan\tfrac{1}{2}\theta_L) + 2\sigma(\theta - \theta_L) \tag{10}$$

If one retains the centrifugal term, one must solve the integrodifferential equation

$$\frac{ds}{d\theta}\sin^2\theta + \int_{\theta_L}^{\theta} \cos\theta' \sin\theta' \frac{ds}{d\theta'} d\theta' = (T/2q_\infty)(1 + 2\sigma \sin\theta) \tag{11}$$

Putting $\phi(\theta) = (ds/d\theta)\sin^2\theta$ and differentiating Eq. (11), one finds

$$\phi'(\theta) + \cot\theta\, \phi(\theta) = (T/2q_\infty)2\sigma \cos\theta \tag{12}$$

which has the solution

$$\phi(\theta)\sin\theta = \frac{T}{2q_\infty}\sin\theta_L (1 + \sigma \sin\theta_L) + \sigma \frac{T}{2q_\infty}\sin^2\theta \tag{13}$$

for, from Eq. (11), when $\theta = \theta_L$, $\phi(\theta_L) = (T/2q_\infty)(1 + 2\sigma \cdot \sin\theta_L)$.

Thus one finds

$$\frac{2q_\infty}{T} s = \sin\theta_L(1 + \sigma \sin\theta_L) \int_{\theta_L}^{\theta} \frac{d\theta}{\sin^3\theta} + \sigma \int_{\theta_L}^{\theta} \frac{d\theta}{\sin\theta} = \tfrac{1}{2}(1 + \sigma \sin\theta_L)\{\cot\theta_L - \cot\theta(\sin\theta_L/\sin\theta) + \sin\theta_L \ln(\tan\tfrac{1}{2}\theta/\tan\tfrac{1}{2}\theta_L)\} + \sigma \ln(\tan\tfrac{1}{2}\theta/\tan\tfrac{1}{2}\theta_L) \tag{14}$$

$$2q_\infty x/T = \tfrac{1}{2}(1 + \sigma \sin\theta_L)[(1/\sin\theta_L) - (1/\sin\theta)\cdot(\sin\theta_L/\sin\theta)] + \sigma \ln(\sin\theta/\sin\theta_L) \tag{15}$$

$$2q_\infty y/T = (1 + \sigma \sin\theta_L)[\cos\theta_L - \cos\theta(\sin\theta_L/\sin\theta)] + \sigma(\theta - \theta_L) \tag{16}$$

The aerodynamic forces and moments on the sail follow immediately from resolving and taking moments of the attaching forces, and are simply expressed in terms of the sail geometry.[2-5]

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Mechanism of the Accelerated Burning of Ammonium Perchlorate at High Pressures

O. R. Irwin,* P. K. Salzman,† and W. H. Andersen‡
Aerojet-General Corporation, Downey, Calif.

The linear burning rate of ammonium perchlorate previously has been shown to undergo a marked increase in pressure dependence at high pressures. The effect was considered to result from an increased burning surface area formed by a shear process at the burning surface as a result of the high pressure. The possibility that the steep thermal gradient existing at the burning surface at high pressures also could lead to the shear stress responsible for cracking is investigated. The analysis indicates that thermal stress is almost solely responsible for the cracking over the entire pressure range of the burning rate experiments.

THE deflagration characteristics of pure ammonium perchlorate (AP) strands previously have been investigated by means of a closed-bomb strand burning technique at pressures from 1000 to 23,000 psi.[1] The data are in general agreement with vented-chamber AP burning-rate data of other investigators at pressures from 1000 to 5000 psi. At pressures above 5000 psi (the pressure limit of previously reported studies), a marked increase in pressure dependence of the linear burning rate occurs.

The observed increase in burning rate was considered to result from an increased burning surface area produced by the action of the very high pressures in the closed bomb.[1] It was postulated that the pressure (i.e., stress) acting upon the nonhomogeneous burning surface caused shearing that gave rise to increased burning area by forming new cracks and pores or by enlarging existing cracks and pores. The crack growth process was analyzed in terms of the Eyring theory of creep and fracture. A geometrical model was presented which considers the accelerated burning process as a development of micro-cracks that form into conically shaped burning surfaces, the area of which depends upon the pressure. The model was in good agreement with the experimental burning-rate data and with the pressure vs time data for individual burning-rate experiments at pressures above 5000 psi.

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* Research Chemist, Ordnance Division. Member AIAA.

† Research Engineer, Ordnance Division.

‡ Technical Specialist, Ordnance Division.

Although the basic assumption of the model that accelerated burning results from a stress-induced formation of new burning surface appears plausible, certain doubts exist as to the nature of the process that causes the stress responsible for crack growth. It has been suggested[2] that a stress might be induced by thermal expansion of the inhomogeneous pressed solid when the hot "combustion wave" propagates into it. The stress involved would be related to the temperature gradient, which would increase with burning rate and hence with ambient pressure.

The suggestion that thermal, rather than mechanical, effects may play a dominant role in deflagration-induced crack propagation is attractive, particularly in the case of a brittle material such as AP. In support of such a view are the observations of Bowden, Yoffe, and their co-workers[3] and of Patry and Lafitte[4] on cracking accompanying the deflagration of crystals at low pressures. (Even in these instances, however, the possibility exists that the pressure in such cracks may differ appreciably from ambient pressure.) The observed cracking of sapphire crystals during high-speed sliding-friction experiments has been analyzed quantitatively by Miller[5] in terms of the thermal stresses produced by the high temperatures in the contact area.

To test the thermal cracking hypothesis, the relative contributions of the shear stresses induced by the thermal gradient and by pressure at various pressure levels are estimated as follows. The variation of burning surface temperature T_s with pressure may be found by combining a relationship[6] for the linear pyrolysis rate of AP,

$$B = 12.2T_s \exp(-22{,}000/RT_s) \text{ in./sec} \tag{1}$$

with the expression for the linear surface regression rate (i.e., the normal burning rate r_{b1}),

$$r_{b1} = 0.065P^{0.251} \text{ in./sec} \tag{2}$$

to give

$$12.2T_s \exp(-22{,}000RT_s) = 0.065\mathrm{P}^{0.251} \tag{3}$$

Equation (3) was solved iteratively for T_s vs P on an IBM 1620 computer. The resulting data were used to compute an assumed temperature profile beneath the burning AP surface at a number of pressures by means of an expression[7] for the effect of moving heat sources upon the temperature distribution in solids:

$$(T_x - T_0)/(T_s - T_0) = \exp(-vx/k_d) \tag{4}$$

where T_x is the temperature at a distance x beneath the hot surface, T_s and T_0 are the temperature of the burning surface and ambient temperature, respectively, k_d is the thermal diffusivity of the solid, and v is the velocity of the moving heat source (i.e., the burning surface); this velocity is considered to be the normal burning rate r_{b1}. The temperature gradient at a point x beneath the surface is then

$$\left(\frac{dT}{dx}\right)_x = -\frac{(T_s - T_0)v}{k_d} \exp\left(\frac{-vx}{k_d}\right) \tag{5}$$

Equation (4) was used to calculate the temperature profiles in burning AP at pressures of 5000, 10,000, and 20,000 psi (Fig. 1), and the (maximum) thermal gradient at the AP surface (i.e., $x = 0$) was computed at various pressures (Fig. 2), with $k_d = 2.92 \times 10^{-4}$ in.²/sec and $T_0 = 300$°K, using Eq. (5).

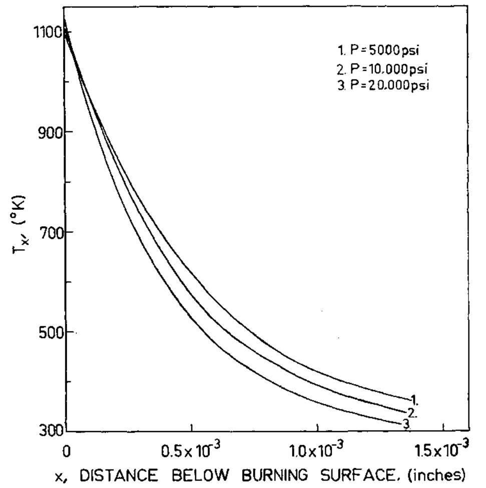

Fig. 1 Temperature profile beneath burning ammonium perchlorate surface at various pressures

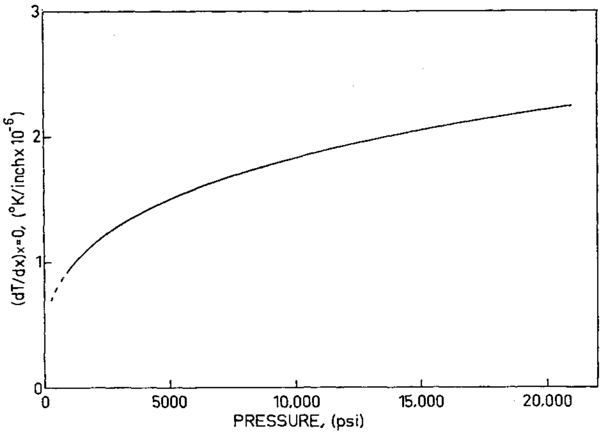

Fig. 2 Effect of pressure on the thermal gradient at the surface of burning ammonium perchlorate

In the previously reported model,[1] the chamber pressure was considered responsible for crack formation and/or propagation through an associated shear stress σ_P related to the pressure (i.e., compressive stress) by the expression

$$\sigma_P = P/n \tag{6}$$

A similar expression can be written for the relationship between the thermal gradient and the shear stress σ_T generated by a thermal expansion[5] for the case of one-dimensional heat conduction:

$$\sigma_T = (\phi G l)\ dT/dx \tag{7}$$

where G is the shear modulus, l the length of the specimen, and ϕ the coefficient of thermal expansion. Assuming that the total shear stress is the sum of the shear stresses due to the thermal gradient and the pressure, an estimate of the magnitudes of σ_T and σ_P gives an indication of the relative contributions of these stresses to crack propagation. For the case of σ_P, the factor n will not differ appreciably from unity so that the magnitude of σ_P may be considered essentially equal to the pressure. To estimate σ_T, the previously calculated values of $(dT/dx)_{x=0}$ (Fig. 2) must be combined with the term $(\phi G l)$. Although the required data for AP are not available, the values used by Miller[5] of ϕ, G, and l for sapphire at high temperature permit a crude estimate to be made. In the given units, the $(\phi G l)$ term of Eq. (7) is also essentially unity, so that the relative contributions of σ_T and σ_P to the total stress are given directly by the magnitudes of $(dT/dx)_{x=0}$ and P, respectively. The results of this calculation are shown in Fig. 3.

The results indicate, in agreement with the suggestion of Friedman,[2] that the thermally induced stress σ_T is almost solely responsible for crack propagation over the entire pressure range of the burning-rate experiments. The use of the actual values for the temperature-dependent "constants" G, ϕ, n, and k_d for AP conceivably might alter the absolute magnitudes of σ_T and σ_P, but it is unlikely that the ratio

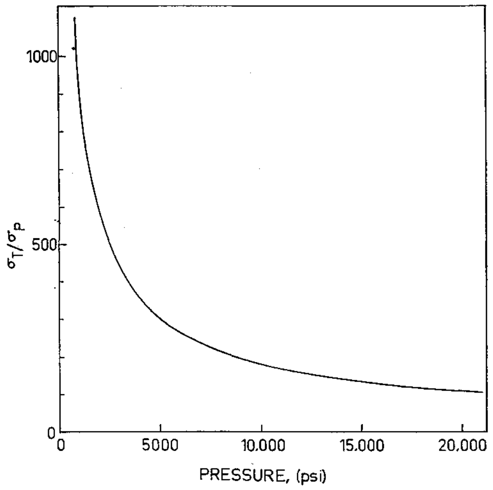

Fig. 3 Ratio of thermal stress to pressure stress at the surface of burning ammonium perchlorate as a function of pressure

σ_T/σ_P could be lowered sufficiently to make the effect of σ_P on cracking predominant.

It should be noted that in the previously proposed model[1] the expression for cracking velocity was

$$v = 2\alpha \sinh \beta P \tag{8}$$

where the constant β contains the arbitrary conversion constant n relating chamber pressure to the shear stress [Eq. (6)], which is responsible for cracking. The present analysis indicates that this stress arises from the pressure-dependent thermal gradient rather than from the pressure itself. Since the thermal stress is nearly linear in pressure for $P > 5000$ psi (Fig. 2), the model essentially is unaffected.

Certain factors not considered in the present treatment also may affect the cracking process. Crystal phase changes, with associated possible changes in crystal density (e.g., the change of orthorhombic to cubic AP at 240°C), may be involved in the cracking. This effect has been observed experimentally by Bowden and McAuslan in the thermal decomposition of silver azide.[3] Also, if the deflagrating material melts during decomposition, cracking may be *prevented*, either by relieving thermal stresses at the burning surface or by preventing penetration of hot gases into existing pores. The latter mechanism has been postulated by Taylor[8] to explain the absence of accelerated burning at low pressures in the burning of porous HMX charges.

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Initial Behavior of a Gun-Tunnel Piston

Takeo Sakurai*
Kyoto University, Kyoto, Japan

RECENTLY, the gun-tunnel has been used widely for the investigation of hypersonic flow. As is well known, it produces hypersonic flow with very high stagnation temperature by means of a piston motivated by the pressure difference between the high- and the low-pressure chambers. The behavior of the piston is essential to the performance of the apparatus, so that many studies of it have been made. Because of the mathematical difficulties, however, there is no analytical expression for the initial behavior of the piston. Hence, it will be interesting to obtain such an expression even for a very extreme condition. In this note the initial behavior of the gun-tunnel piston with a very small pressure difference is investigated by the small-perturbation method.

An infinitely long straight channel is divided by a diaphragm into high- and low-pressure chambers, and a piston is placed close to the diaphragm in the low-pressure chamber. The problem here is to discuss the behavior of the piston after bursting of the diaphragm within the small-perturbation method and without considering friction between the piston and the channel walls.

The basic equations of the one-dimensional unsteady motion of a compressible perfect fluid within the small-perturbation approximation are as follows:

$$P_i = P_{i0} + P_i' \qquad \rho_i = \rho_{i0} + \rho_i' \qquad U_i = u_i \tag{1}$$

$$P_i' = -\rho_{i0}\frac{\partial \varphi_i}{\partial t} \qquad \rho_i' = \frac{P_i'}{C_{i0}^2} \qquad u_i = \frac{\partial \varphi_i}{\partial x} \tag{2}$$

$$0 = [(\partial^2/\partial t^2) - C_{i0}^2(\partial^2/\partial x^2)]\varphi_i \tag{3}$$

where P, ρ, U, t, x, and C indicate the pressure, density, velocity, time, position, and sound speed, respectively. The subscripts 1, 2, and 0 indicate quantities in the high-pressure chamber, the low-pressure chamber, and the undisturbed state, respectively.

The initial conditions at the instant the diaphragm bursts are

$$P_1' = P_2' = 0 \qquad U_1 = U_2 = 0 \tag{4}$$

The boundary conditions on the piston are

$$u_1|_{x = f(t) - 0} = u_2|_{x = f(t) + 0} = f'(t) \tag{5}$$

$$mf'' = P_1 - P_2 \tag{6}$$

where m is the mass of the piston and $x = f(t)$ describes the locus of the piston in the (x, t) plane. The last equation is simply the equation of motion for the piston.

The general solution of the wave equation (3) can be obtained as follows:

$$\varphi_i = F_i(x + C_{i0}t) + G_i(x - C_{i0}t) \tag{7}$$

Substituting (7) into the initial conditions (4), it can be shown that φ_1 and φ_2 are both zero below the wave front in Fig. 1, i.e., the fluid in both the high- and the low-pressure chambers is not disturbed until the wave front reaches it. Then, by the continuity of the potential on the wave front, the solution in the region between the wave front and the piston can be obtained as follows:

$$\varphi_1 = F_1(x + C_{10}t) \tag{8}$$

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* Assistant Professor, Department of Aeronautical Engineering, Faculty of Engineering.